Economics 241B
Endogeneity Bias - The Example of Working

The classic illustration of the biases created by endogeneity dates to Working in 1927. Consider the simple model of demand and supply (for coffee, say)

$$\begin{aligned} Q_t^d &= \alpha_0 + \alpha_1 P_t + U_t \\ Q_t^s &= \beta_0 + \beta_1 P_t + V_t \\ Q_t^d &= Q_t^s. \end{aligned}$$

The error term U_t in the demand equation captures factors that influence coffee demand, other than price, such as the public's mood for coffee. Depending on the value of U_t, the demand curve in the price-quantity plane shifts up or down. Similarly, V_t represents supply factors other than price. As the intercepts capture the mean of the error terms, we assume $E(U_t) = 0$ and $E(V_t) = 0$. To avoid inessential complications, we also assume $Cov(U_t, V_t) = 0$. If we define the market-clearing quantity to be $Q_t = Q_t^d = Q_t^s$, then the system reduces to two equations

$$\begin{aligned} Q_t &= \alpha_0 + \alpha_1 P_t + U_t \quad \text{(demand)} \\ Q_t &= \beta_0 + \beta_1 P_t + V_t. \quad \text{(supply)} \end{aligned}$$

A regressor is endogenous if it is not orthogonal to the error term, that is, if it does not satisfy the orthogonality condition. With an intercept in the equation, endogeneity arises if and only if the regressor is correlated with the error. In the present example, the regressor P_t is necessarily endogenous in both equations. To see why, treat the demand and supply equations as a system of two simultaneous equations and solve for (P_t, Q_t) as

$$\begin{aligned} P_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{V_t - U_t}{\alpha_1 - \beta_1} \\ Q_t &= \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 V_t - \beta_1 U_t}{\alpha_1 - \beta_1}. \end{aligned}$$

Because price is a function of the two error terms, price is clearly correlated with each error.

We can take the argument one step further and calculate the covariance of price with each error. The covariance of price with the demand shifter U_t and the supply shifter V_t is

$$Cov\left(P_t, U_t\right) = -\frac{Var\left(U_t\right)}{\alpha_1 - \beta_1}, \quad Cov\left(P_t, V_t\right) = \frac{Var\left(V_t\right)}{\alpha_1 - \beta_1}.$$

If the demand curve is downward sloping ($\alpha_1 < 0$) and the supply curve is upward sloping ($\beta_1 > 0$), then price is correlated positively with the demand shifter and negatively with the supply shifter.

Endogeneity Bias

When quantity is regressed on price, neither the demand curve nor the supply curve is estimated. To understand what the OLS coefficient estimates do capture, recall the coefficient from a least squares projection of Q_t on a constant and P_t is, by definition, $Cov\left(P_t, Q_t\right)/Var\left(P_t\right)$. Hence, for the OLS estimate

$$\text{OLS estimate of the coefficient on price } \underset{p}{\rightarrow} \frac{Cov\left(P_t, Q_t\right)}{Var\left(P_t\right)}.$$

To rewrite this ratio in terms of the price effect in the demand curve (α_1), the demand equation yields

$$Cov\left(P_t, Q_t\right) = \alpha_1 Var\left(P_t\right) + Cov\left(P_t, U_t\right).$$

Substituting this expression into the probability limit for the OLS estimate yields the asymptotic bias for α_1

$$\text{plim of the OLS estimate of the price coefficient } - \alpha_1 = \frac{Cov\left(P_t, U_t\right)}{Var\left(P_t\right)}.$$

Similarly, the asymptotic bias for β_1, the price effect in the supply curve, is

$$\text{plim of the OLS estimate of the price coefficient } - \beta_1 = \frac{Cov\left(P_t, V_t\right)}{Var\left(P_t\right)}.$$

The bias of the estimated coefficient for the price effects is referred to as simultaneity bias, a special case of the more general phenomenon of endogeneity bias. Clearly the bias vanishes if the correlation between the price regressor and the shifter is zero. Thus, if $U_t = 0$ for all t, then the demand shifter vanishes (only the supply curve shifts-**draw figure**), then the variance of the demand shifter is zero, which implies that price is not correlated with the demand shifter and the demand curve coefficient (α_1) can be consistently estimated. In parallel fashion, if only the demand curve shifts, then the parameters of the supply curve can be consistently estimated. As both curves generally shift, the OLS estimate provides a consistent estimate of a linear combination of the demand and supply curve parameters

$$\text{plim of the OLS estimate of the price coefficient} = \frac{\alpha_1 Var(V_t) + \beta_1 Var(U_t)}{Var(V_t) + Var(U_t)}.$$

Clearly, the larger is the variation in the supply shifter (V_t) the closer is the probability limit of the estimate to the demand curve coefficient. Hint, to establish this recall $Cov(U_t, V_t) = 0$ so $Var(P_t) = \frac{1}{(\alpha_1-\beta_1)^2}(Var(V_t) + Var(U_t))$ and $Cov(P_t, Q_t) = \frac{1}{(\alpha_1-\beta_1)^2}(\alpha_1 Var(V_t) + \beta_1 Var(U_t))$.

Observable Supply Shifters

The reason that neither the demand curve nor the supply curve can be consistently estimated is that we cannot infer from the data whether the change in price and quantity is due to a demand shift or a supply shift. If it is possible to observe factors that are known to shift the curves, then it may be possible to consistently estimate the parameters of demand and supply. To illustrate, consider the supply shifter V_t. If the temperature in the coffee-growing regions affects supply but not demand for coffee (as seems reasonable), the temperature forms an observable shifter of supply. If we let X_t measure the temperature, then we rewrite the supply equation as

$$Q_t = \beta_0 + \beta_1 P_t + \beta_2 X_t + \zeta_t,$$

where ζ_t is the unobservable component (that is, the part of V_t uncorrelated with X_t) of the supply shifter. (Note, this decomposition is always possible. The least squares projection of V_t on a constant and X_t is $\gamma_0 + \beta_2 X_t$, so $\zeta_t = V_t - (\gamma_0 + \beta_2 X_t)$ is uncorrelated with X_t.)

As the temperature in coffee-growing regions is uncorrelated with the demand shifter U_t by assumption, and uncorrelated with ζ_t by construction, it is an instrumental variable (or simply an instrument). As the temperature is correlated with price, it is a valid instrument. (Note, because quantity is a function of X_t in the supply equation, it is the case that (when solving the two equations for) P_t is a function of X_t.) We return to the demand equation and use X_t as an instrument

$$Cov\left(X_t, Q_t\right) = \alpha_1 Cov\left(X_t, P_t\right) + Cov\left(X_t, U_t\right).$$

Because $Cov\left(X_t, U_t\right) = 0$ by assumption, $\alpha_1 = \frac{Cov(X_t,Q_t)}{Cov(X_t,P_t)}$ and the corresponding method of moments estimator is

$$\hat{\alpha}_{1,IV} = \frac{\text{sample covariance between } X_t \text{ and } Q_t}{\text{sample covariance between } X_t \text{ and } P_t}.$$

In forming this instrumental variables estimator, we sometimes say that the endogenous regressor P_t is instrumented by X_t.

Another popular estimator for α_1 is the two-stage least squares (2SLS) estimator. The two stages refer to two regressions. In the first stage, P_t is regressed on a constant and the instrument to obtain the fitted value $\hat{P}_t$. In the second stage, Q_t is regressed on a constant and $\hat{P}_t$. Thus

$$\hat{\alpha}_{1,2SLS} = \frac{\text{sample covariance between } X_t \text{ and } \hat{P}_t}{\text{sample variance of } \hat{P}_t}.$$

To understand the difference between the second stage regression and simple OLS, which is inconsistent, note the demand equation is

$$Q_t = \alpha_0 + \alpha_1 \hat{P}_t + \left[U_t + \alpha_1 \left(P_t - \hat{P}_t\right)\right].$$

The 2SLS estimator of α_1 is obtained by estimating this equation, treating the bracketed term as the error. If the fitted value $\hat{P}_t$ was exactly equal to the least squares projection, then the fitted value would be orthogonal to the bracketed error. In fact, the fitted value differs from the projection because the coefficients from the first stage are estimated, and this induces a slight correlation. Hence the 2SLS estimator is biased, although substantially less biased than is the OLS estimator. Moreover, as the sample size grows the estimated coefficients in the first stage converge to the population coefficients, so the fitted value converges to the least squares projection and the 2SLS estimator is consistent.

Finally, the least squares projection is the best linear predictor, so the fitted value incorporates all the effect of the supply shifter on price. This suggests that the asymptotic variance is minimized for the 2SLS estimator. Moreover, in this case with exact identification, the 2SLS and IV estimators are numerically the same value. In more general cases, the 2SLS estimator can always be written as an IV estimator (with the appropriate choice of instruments) and the IV estimator, in turn, is a particular GMM estimator.